Dynamical Modelling for Flat-Spin Recovery Applications

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The recovery from a flat-spin motion is a remarkable practical application of spinning-satellite dynamics.[1-2] Flat-spin refers to a satellite that initially spins about its *minimum* axis of inertia but under energy-dissipation effects, ends up spinning about its *maximum* axis of inertia (since this is the minimum-energy state). A flat-spin recovery re-establishes the intended *minimum*-inertia spin.

If a body-fixed torque acts about a principal axis of inertia there exists a linear combination of energy E and angular momentum-squared H^2 that is a first integral of motion. For a torque about the minimum axis of inertia axis (x), this integral is denoted by ΔE_{max}, which is the deficit between the current value of the energy $E(t)$ and its maximum possible value E_{max} for a given value of $H(t)$:

$$\Delta E_{max}(t) = \frac{H^2(t)}{2A} - E(t) = \frac{BC}{2A}\left\{k_3\omega_2^2(t) + k_2\omega_3^2(t)\right\} \geq 0 \tag{1}$$

$A < B < C$ are the moments of inertia along x, y, z and ω_j ($j = 1, 2, 3$) are the rates about x, y, z, and:

$$k_1 = (C - B)/A;\quad k_2 = (C - A)/B;\quad k_3 = (B - A)/C \tag{2a-c}$$

Eq. (1) shows that ω_2, ω_3 move on an ellipse with varying semi-major and minor-axes $a(t)$ and $b(t)$:

$$\omega_2(t) = a(t)\sin\varphi(t);\quad \omega_3(t) = b(t)\cos\varphi(t) \tag{3}$$

with:

$$a(t) = \sqrt{\frac{2A}{BC}\frac{\Delta E_{max}}{k_3}} > b(t) = \sqrt{\frac{2A}{BC}\frac{\Delta E_{max}}{k_2}} \Rightarrow b = \kappa a \quad with:\ \kappa = \sqrt{\frac{k_3}{k_2}} \tag{4a-d}$$

The angle $\varphi(t)$ in Eq. (3) is the eccentric anomaly of the osculating ellipse.

Finally, we propose a new formulation using the variables $a(t)$ and $\varphi(t)$ instead of ω_2 and ω_3:

$$\begin{aligned} \dot{\omega}_1(t) &= m_1 - \kappa k_1 a^2 \sin\varphi\cos\varphi \\ \dot{a}(t) &= m_2\sin\varphi(t) + (m_3/\kappa)\cos\varphi(t) \\ \dot{\varphi} &= k_n\omega_1 + \{m_2\cos\varphi - (m_3/\kappa)\sin\varphi\}/a \quad with:\ k_n = \sqrt{k_2 k_3} \end{aligned} \tag{5a-d}$$

After the recovery is achieved, the spin rate $\omega_1(t)$ oscillates about a straight line, see Fig. 1. Eq. (5b) leads to compact analytical solutions in terms of Fresnel Integrals. Their known asymptotic properties enable us to predict the asymptotic values of ΔE_{max} and the nutation angle. Fig. 2 shows the asymptotic results for ΔE_{max} for a torque with a fixed T_1= 10 Nm and a range of T_2 values.

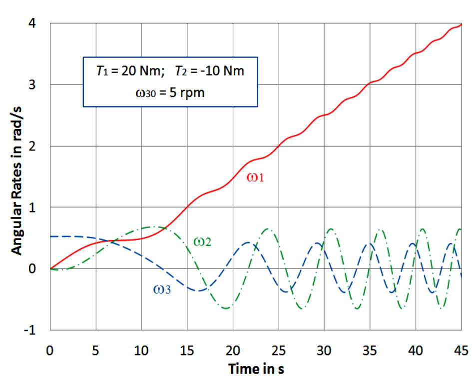

Fig. 1. Example of Fast Flat-Spin Recovery.

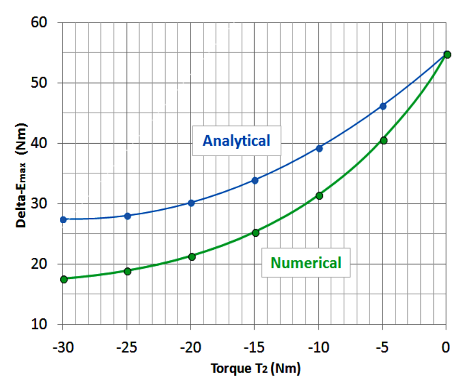

Fig. 2. $\Delta E_{max}(T_2)$ with Fixed T_1 = 20 Nm.

References

[1] F.L. Janssens and J.C. Van der Ha, Flat-Spin Recovery of Spinning Satellites by an Equatorial Torque, *Acta Astronautica*, **116** (2015), 355-367.

[2] F.L. Janssens and J.C. Van der Ha, Analytical Solution for Flat-spin Recovery of Spinning Satellites, *Advances in Astronautical Sciences, Space Flight Mechanics*, **152** (2014), 3495-3513.